



INCREASING RETURNS AND COMPLEXITY

HSIN PING CHEN

Department of Economics, National Chengchi University, Taipei, Taiwan.

E-mail: spchen@nccu.edu.tw

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Abstract: Dynamic economic processes influenced by random events and positive feedbacks deviate significantly from classical equilibrium models. They often exhibit multiple equilibria, path dependence, and lock-in, with outcomes strongly shaped by early events. Path dependence and feedback strength critically influence long-term outcomes. Increasing returns systems exhibit self-reinforcing behavior, where small early leads can become dominant outcomes. These systems often have multiple equilibria, where outcomes are contingent, not inevitable. In contrast, decreasing returns processes tend toward single equilibrium (balance), reducing path dependence. Processes with threshold-like or stepwise reinforcement show extreme sensitivity, behaving almost deterministically based on early conditions.

Keywords: positive feedback, increasing returns, nonlinear process

1. INTRODUCTION

Neoclassical economics is based on several core assumptions. It assumes rational agents who maximize utility or profits. Equilibrium-based models where markets tend to settle in stable states. Representative agents that simplify heterogeneous behavior into an “average” decision-maker and perfect information with market-clearing behavior. These assumptions allow for elegant mathematical modeling but often fall short in explaining systemic crises, persistent unemployment, or innovation dynamics.

Complexity Economics views the economy as an adaptive system composed of heterogeneous agents whose interactions produce emergent macro-level

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phenomena. Agents vary in preferences, information, strategies, and resources. This heterogeneity drives diverse interactions and adaptive learning. Economic outcomes emerge from local interactions in networks (e.g., firms, households, governments). These interactions are often nonlinear and produce emergent behavior. Small changes can lead to disproportionate outcomes (e.g., a single bank failure triggering a systemic crisis).

Positive feedbacks result in increasing returns features in some markets such as tech, economic geography and internal trade. Path-dependent meaning small changes in initial conditions (e.g., policy shocks) can lead to vastly different macroeconomic outcomes. Economic interactions occur within complex networks (e.g., trade, finance, or supply chains). Path dependency and network effects, which can lead to lock-in and systemic vulnerabilities (Arthur, 1994). Macro outcomes emerge from micro-level behaviors without central coordination (Arthur, 2009; Beinhocker, 2006). The system dynamically evolving rather than gravitating toward a single equilibrium. Innovation and novelty are endogenous to the system (Beinhocker, 2006).

2. DYNAMIC PROCESSES WITH RANDOM EVENTS AND POSITIVE FEEDBACKS

Dynamic processes involving random events and positive feedbacks exhibit complex and often nonlinear behavior. Random events (or stochastic shocks) introduce uncertainty into the system, while positive feedbacks amplify the effects of initial conditions or small advantages, potentially leading to disproportionately large outcomes over time. These processes are characterized by nonlinearity, sensitivity to initial conditions, Path dependence, amplification of small random fluctuations, and Multiple equilibria. Early small advantages or disadvantages can be magnified over time, potentially leading to self-reinforcing behavior. The eventual outcome is strongly influenced by the historical sequence of events, not merely by current conditions. And the results are Sensitive to initial conditions Small differences at the outset can lead to divergent long-term outcomes.

Positive feedback implies that a success or gain in the past increases the probability of further success in the future. In contrast to systems with diminishing returns that stabilize over time, systems with increasing returns become more unstable and path-dependent. These elements challenge

traditional equilibrium analysis and call for evolutionary and complexity-based modeling (Arthur, Durlauf & Lane, 1997).

Markets with increasing returns are characterized by that the value or efficiency of a product or technology increases with the number of users. For examples in Technology markets, Search engines markets, and Social media networks.

The dominance of Microsoft Windows in the 1990s illustrates increasing returns due to network effects—the more people used Windows, the more developers created compatible software, further attracting users (Arthur, 1989). Platforms like Facebook or TikTok benefit from network effects—more users attract more content creators, enhancing platform value. Search engines like Google's increasing market share allowed it to gather more data, improve algorithms, and attract more users and advertisers—a classic feedback loop. VHS over Betamax in video technology was influenced by small initial advantages, which were amplified by positive feedbacks such as market share and production scale.

Determinacy and Equilibrium in Dynamic Processes with Feedbacks

Unlike neoclassical models with unique equilibria and stable outcomes, dynamic systems with positive feedback and stochastic shocks are often indeterminate. They do not necessarily converge to a unique equilibrium; instead, they can settle into one of multiple possible equilibria, depending on random historical events (Arthur, 1989; Krugman, 1991). With increasing returns and stochasticity, equilibrium may be non-unique and history-dependent. This results from nonconvexities in production or utility functions, making the standard equilibrium existence theorems inapplicable (Arthur, 1989; Krugman, 1991).

Randomness (stochasticity) influences early outcomes, which are then locked in through feedback mechanisms. This process gives rise to path dependence, where history matters, and small, possibly random events can have long-term impacts.

Systems become locked-in to particular paths, not necessarily the most efficient or optimal. The system may be irreversibly committed to a specific trajectory, even if suboptimal compared to alternatives. For instance, QWERTY keyboards are a classic case of historical lock-in (David, 1985), where early adoption and coordination led to widespread use, despite arguably

less efficient alternatives. Once an Operating system gains a dominant market share, switching costs and compatibility issues lock users into that system.

3. NONLINEAR STOCHASTIC PROCESSES AND INCREASING RETURNS

The mathematical modeling of these systems often employs nonlinear stochastic differential equations (SDEs) or Markov processes with absorbing states.

To formalize these dynamics, we turn to nonlinear stochastic processes, such as Markov chains with absorbing states and stochastic differential equations with non-convex potentials. Small changes can have disproportionately large effects. may play a significant role in determining the trajectory. It results in Reinforcement mechanisms it is often modeled via polya urn processes, where the probability of an outcome increases with its past occurrence.

Power laws describe a statistical relationship where the frequency of an event scales as a power of some attribute of that event (e.g., size or frequency). This type of distribution has been widely observed in systems with increasing returns, such as: Firm sizes, Wealth distribution (Pareto distributions), and City sizes (Zipf's law). A small number of firms dominate markets, consistent with power law tails. Zipf's Law suggests the size of a city is inversely proportional to its rank. Pareto's Law observes that a small percentage of the population controls a large percentage of wealth. Power laws emerge from processes involving preferential attachment or cumulative advantage, closely related to positive feedbacks. Entities that are already successful have a higher chance of gaining even more, leading to heavy-tailed distributions.

A model of nonlinear Polya urn process is applied to generate path dependence processes. We explore the dynamic stochastic processes with different degrees of feedback to explain the features of increasing returns in dynamic process. The Polya urn process is a classic example of a stochastic process exhibiting path dependence, where past outcomes influence future probabilities. The basic (linear) version of the process assumes reinforcement is proportional to the existing number of balls. By introducing a nonlinearity parameter r , we can model both increasing returns ($r > 1$) and decreasing returns ($r < 1$) dynamics. The probability of drawing a red ball is:

$$P(R_t) = R_t^r / (R_t^r + B_t^r)$$

Where R_t is number of red balls at time t . B_t is number of black balls at time t . The value of coefficient r indicates the degree and types of increasing return, decreasing return or constant return.

We simulate and explore 10 different levels of Polya urn process (5 with increasing returns features and 5 with decreasing returns) and 5 linear dynamic stochastic process (each dynamic stochastic process given 30 different initial conditions or coefficient). The figures show the dynamic process by corresponding coefficients. Figure 1 shows the distribution of final red proportion by process type and reinforcement coefficient. Figure 2 shows the time path of red ball proportions for each 15 Dynamic Stochastic Processes.

Figure 1 shows that when r is greater than one indicating increasing return: the larger the value of r , the stronger the positive feedback the smaller the variance of the distribution is. when r is smaller than one indicating decreasing return: the smaller the value of r , it shows the stronger the decreasing returns, the smaller the variance of the distribution. Figure 2 shows that when r equals one, which is linear return. Outcome is random but still allows for long-term dominance. When r is greater than one, which is increasing return. Path dependence becomes more pronounced. A small early advantage leads to lock-in and irreversibility. The larger the value of r , the more deterministic the dominance becomes. When r is smaller than one, which is decreasing return. Even if one color is initially ahead, there's a stronger pull back to balance. Less

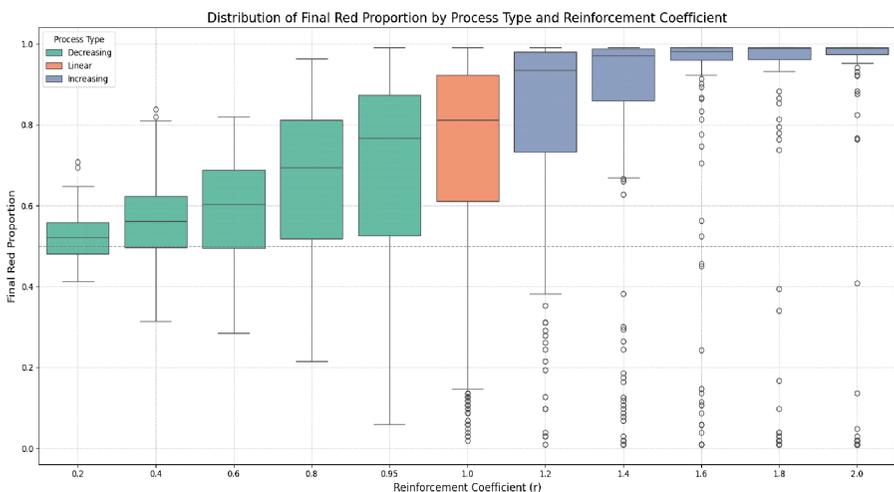


Fig. 1: The distribution of final red proportion by process type and reinforcement coefficient

Time Path of Red Ball Proportions
for 15 Dynamic Stochastic Processes

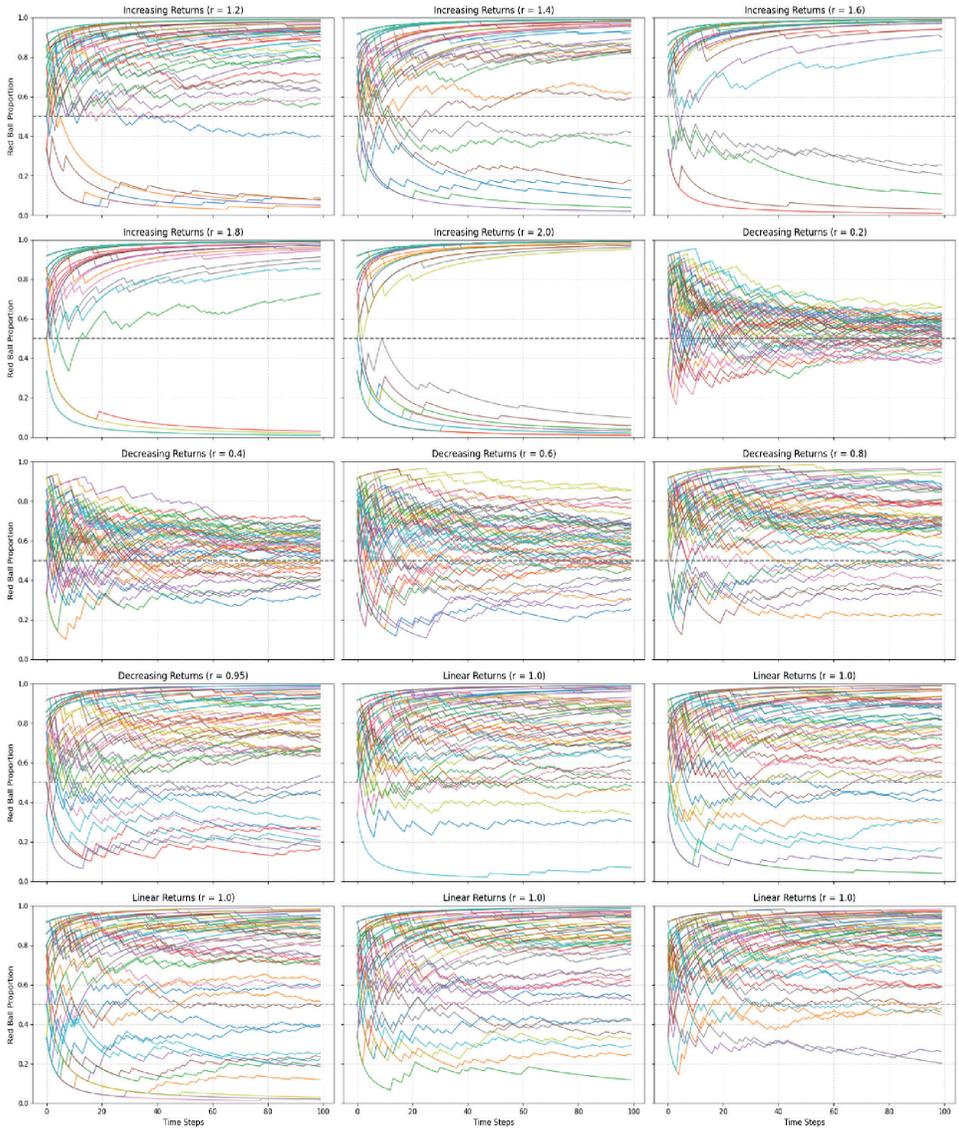


Fig. 2: The time path of red ball proportions for each 15 Dynamic Stochastic Processes

sensitive to initial fluctuations. The smaller the value of r , the process more tends to stabilize near 0.5; the less important the initial conditions are; and the average value of the distribution is closer to 0.5.

4. CONCLUDING REMARKS

Dynamic economic processes influenced by random events and positive feedbacks deviate significantly from classical equilibrium models. They often exhibit multiple equilibria, path dependence, and lock-in, with outcomes strongly shaped by early events. Theoretical tools from nonlinear stochastic processes and insights into power law distributions provide a robust framework for understanding these systems. Recognizing these properties allows for better policy-making in contexts ranging from technology adoption to industrial organization and market regulation. Complexity economics provides a more realistic, adaptive, and network-aware framework for understanding economic phenomena. Its growing adoption in policy analysis, crisis modeling, and urban planning signals a profound shift in economic thinking. Path dependence and feedback strength critically influence long-term outcomes. Increasing returns systems exhibit self-reinforcing behavior, where small early leads can become dominant outcomes. These systems often have multiple equilibria, where outcomes are contingent, not inevitable. In contrast, decreasing returns processes tend toward single equilibrium (balance), reducing path dependence. Processes with stepwise reinforcement show extreme sensitivity, behaving almost deterministically based on early conditions.

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